magnitude of output voltage results from two very different input displacements. The direction of displacement may be determined by using the fact that there is a 180° phase shift when the core passes through the null position. A phase-sensitive demodulator is used to determine the direction of displacement. Figure 3.17 shows a ring-demodulator system that could be used with the LVDT.

2.5 CAPACITIVE SENSORS

The capacitance between two parallel plates of area A separated by distance x is

$$C = \varepsilon_0 \varepsilon_r \frac{A}{x} \tag{2.8}$$

where ε_0 is the dielectric constant of free space (Appendix A.1) and ε_r is the relative dielectric constant of the insulator (1.0 for air) (Bowman and Meindl, 1988). In principle it is possible to monitor displacement by changing any of the three parameters ε_r, A, or x. However, the method that is easiest to implement and that is most commonly used is to change the separation between the plates.

The sensitivity K of a capacitive sensor to changes in plate separation Δx is found by differentiating (2.8).

$$K = \frac{\Delta C}{\Delta x} = -\varepsilon_0 \varepsilon_r \frac{A}{x^2} \tag{2.9}$$

Note that the sensitivity increases as the plate separation decreases.

By substituting (2.8) into (2.9), we can develop an expression showing that the percent change in C about any neutral point is equal to the per-unit change in x for small displacements. Thus

$$\frac{dC}{dx} = \frac{-C}{x} \tag{2.10}$$

or

$$\frac{dC}{C} = \frac{-dx}{x} \tag{2.11}$$

The capacitance microphone shown in Figure 2.9 is an excellent example of a relatively simple method for detecting variation in capacitance (Doebelin, 1990; Cobbold, 1974). This is a dc-excited circuit, so no current flows when the capacitor is stationary (with separation x_0), and thus $v_1 = E$. A change in

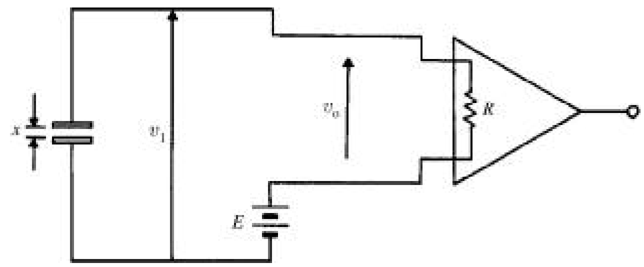


Figure 2.9 Capacitance sensor for measuring dynamic displacement changes.

position $\Delta x = x_1 - x_0$ produces a voltage $v_o = v_1 - E$. The output voltage V_o is related to x_1 by

$$\frac{V_o(j\omega)}{X_1(j\omega)} = \frac{(E/x_0)\, j\omega\tau}{j\omega\tau + 1} \tag{2.12}$$

where $\tau = RC = R\varepsilon_0\varepsilon_r A/x_0$.

Typically, R is 1 MΩ or higher, and thus the readout device must have a high (10 MΩ or higher) input impedance.

For $\omega\tau \gg 1$, $V_0(j\omega)/X_1(j\omega) \cong E/x_0$, which is a constant. However, the response drops off for low frequencies, and it is zero when $\omega = 0$. Thus (2.12) describes a high-pass filter. This frequency response is quite adequate for a microphone that does not measure sound pressures at frequencies below 20 Hz. However, it is inadequate for measuring most physiological variables because of their low-frequency components.

Compliant plastics of different dielectric constants may be placed between foil layers to form a capacitive mat to be placed on a bed. Patient movement generates charge, which is amplified and filtered to display respiratory movements from the lungs and ballistographic movements from the heart (Alihanka *et al,* 1982).

A capacitance sensor can be fabricated from layers of mica insulators sandwiched between corrugated metal layers. Applied pressure flattens the corrugations and moves the metallic plates closer to each other, thus increasing the capacitance. The sensor is not damaged by large overloads, because flattening of the corrugations does not cause the metal to yield. The sensor measures the pressure between the foot and the shoe (Patel *et al.*, 1989). Tsoukalas *et al.* (2006) describe micromachined silicon capacitive sensors and their electronic interfaces.

EXAMPLE 2.1 For a 1 cm^2 capacitance sensor, R is 100 MΩ. Calculate x, the plate spacing required to pass sound frequencies above 20 Hz.

ANSWER From the corner frequency, $C = 1/2\pi fR = 1/(2\pi 20 \times 10^8) =$ 80 pF From (2.8) we can calculate x given the value of C.

$$x = \frac{\varepsilon_0\varepsilon_r A}{C} = \frac{(8.854 \times 10^{-12})(1 \times 10^{-4})}{80 \times 10^{-12}} = 1.11 \times 10^5\ \text{m} = 11.1\ \mu\text{m}$$

designed so that the position at which the measurement is made yields a value representative of the average mass flow through the entire cross section of the duct at every instant of time. At the low Mach numbers involved in respiratory flows, this requires that the velocity profile be well defined at all flows of interest. The variations in cross-sectional area upstream and downstream from the sensor can be optimized for this condition.

Although a single hot-wire sensor provides an output of the same polarity independent of the direction of flow, limiting its use to unidirectional flow, multiple sensors located at separate points along the flow path can be used with the appropriate circuitry to provide directional sensitivity, as described in Section 8.5. However, respiratory measurements involve a changing mixture of gases in the flow stream that affects the heat transfer from the heated wire. The significant variations in composition that occur between inspiration and expiration could invalidate the use of a single calibration factor. During a multibreath N_2 washout (Section 9.4), the N_2–O_2 ratio in the lung changes from approximately 4 to 1 on the first breath to nearly zero at the end of the test. Fortunately, the differences in thermal properties and densities of N_2 and O_2 offset each other sufficiently well that a linearized, temperature-compensated hot-wire anemometer can be used with a constant calibration factor for volume flow measurements during the successive expirations of a multibreath N_2 washout. In general, however, the sensor should be calibrated for the particular gas mixture to which it is exposed.

The hot-wire anemometer has a number of features that are advantageous in respiratory applications. It has an appropriate frequency response (the sensor itself can respond to frequencies into the kilohertz range). Moreover, because the sensing element in the flow stream is extremely small, the only back pressure produced is that caused by the flow through the duct. In unidirectional applications rebreathing does not occur, so dead space is irrelevant. Accurate readings can be made at low as well as high flow rates if the output is adequately linearized. Special circuits can be provided to overheat the sensor to burn off contaminants as necessary. The main disadvantage is the limitation to unidirectional flow. The relatively high cost of overcoming this with a paired hot-wire system may not be justified.

DIFFERENTIAL PRESSURE FLOWMETERS

Convective flow occurs as a result of a difference in pressure between two points. From the relationship between pressure difference and volume flow through a system, measurement of the difference in pressure yields an estimate of flow. Flowmeters based on this idea have incorporated several mechanisms to establish the relationship between pressure drop and flow. These include the venturi, orifice, and flow resistors of various types. A flowmeter based on a modified pitot tube has also been developed.

Venturis and Orifices Venturis and orifices with fixed-sized openings have inherently nonlinear pressure–flow relationships and require calibration charts, special circuitry, or digitally implemented algorithms to be useful as flow sensors. A passive, mechanically linearized orifice flowmeter has been produced in which an elastic flap moved by the pressure of the flowing gas impinging on it increases or decreases the orifice size (Sullivan *et al.*, 1984). However, sensors with computer-controlled orifices that can measure and/or control flow are also in use.

A two-stage, fixed-orifice system has been used to measure flow during forced expiratory vital-capacity maneuvers (Jones, 1990). At high flows, the gas stream passes through a large orifice. When the sensed pressure drop indicates that the flow has decreased to below 2 liter/s, a solenoid is activated, producing a step decrease in the opening and thus increasing the sensor's sensitivity to low flows. Parameters of the measured waveshape are presented on a digital readout.

Pneumotachometers The flow sensors that historically have been, and continue to be, the mainstay of the respiratory laboratory utilize flow resistors with approximately linear pressure–flow relationships. These devices are usually referred to as *pneumotachometers* (Macia, 2006). (In general, the term *pneumotachometer* is synonymous with *gas volume flowmeter.)* Flow-resistance pneumotachometers are easy to use and can distinguish the directions of alternating flows. They also have sufficient accuracy, sensitivity, linearity, and frequency response for most clinical applications. In addition, they use the same differential pressure sensors and amplifiers required for other respiratory measurements. The following discussion primarily concerns these instruments.

Even though other flow-resistance elements have been incorporated in pneumotachometers, those most commonly used consist of either one (Silverman and Whittenberger, 1950) or more (Sullivan *et al.,* 1984) fine mesh screens [Figure 9.3(a)] placed perpendicular to flow, or a tightly packed bundle of capillary tubes or channels [Figure 9.3(b)] with its axis parallel to flow (Fleisch, 1925). These physical devices exhibit, for a wide range of unsteady flows, a nearly linear pressure-drop–flow relationship, with pressure drop approximately in phase with flow.

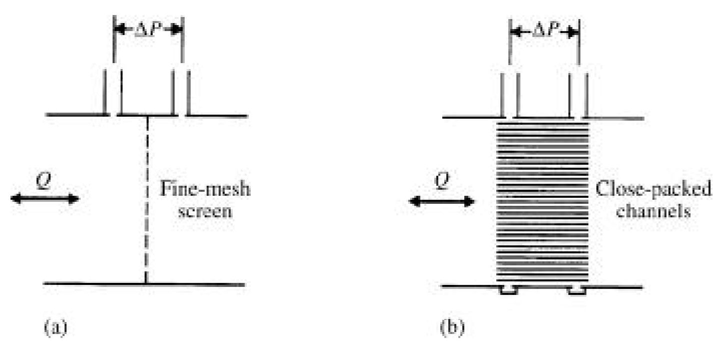


Figure 9.3 Pneumotachometer flow-resistance elements (a) screen and (b) capillary tubes or channels.

In practice, the element is mounted in a conduit of circular cross section. The pressure drop is measured across the resistance element at the wall of the conduit (within the boundary layer of the flow). The pressure tap on each side of the resistance is either a single hole through the conduit wall or multiple holes from a circumferential channel within the wall, which is connected to a common external tap.

Because the pressure drop is measured at a single radial distance from the center of the conduit, it is assumed that this pressure drop is representative of the pressure drop governing the total flow through the entire conduit cross section. Thus these flowmeters rely on the flow-resistive element to establish a consistent—though not uniform—velocity profile on each side of the element in the neighborhood of the pressure measurement. This, however, cannot be achieved independent of the ductwork in which the pneumotachometer is placed (Kreit and Sciurba, 1996). Therefore, the placement of the pressure ports and the configuration of the tubing that leads from the subject to the pneumotachometer and from the pneumotachometer to the remainder of the system are critical in determining the pressure-drop–flow relationship. This is especially important when alternating and/or high-frequency flow patterns are involved.

A number of trade-offs exist in the design and use of these sensors. The ΔP–Q relationship may be more linear for steady flow when there is a large axial separation between pressure ports than when there is a smaller separation. But during unsteady flow with high-frequency content, the pressure drop for a larger separation may be more influenced by inertial forces. If the sensor is to avoid excessive formation of vortices at high flow rates, the cross-sectional area of the conduit at the flow-resistance element must be large enough to reduce the velocity through the element. This area may be several times that of the mouth of the subject from which the gas originates, requiring an adapter or diffuser between the mouthpiece and the resistance element. If flow separation and turbulence are to be avoided as the cross-sectional area changes, the adapter should have a shallow internal angle, θ, [Figure 9.4(a)] not

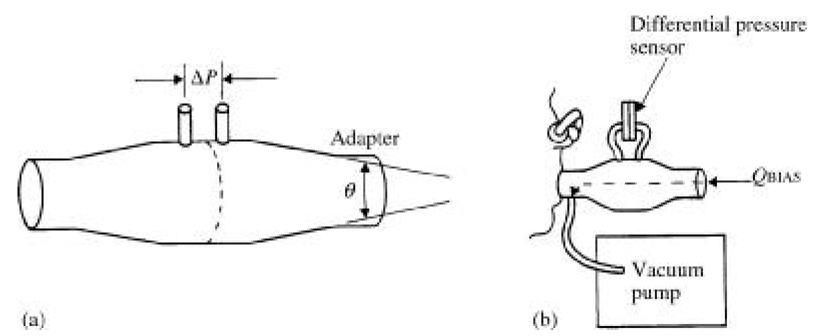


Figure 9.4 Pneumotachometer for measurements at the mouth (a) Diameter adapter that acts as a diffuser. (b) An application in which a constant flow is used to clear the dead space.

exceeding 15°. The shallower the angle, however, the greater is the distance between the mouth and the flow-resistance element. Symmetric drops in pressure for the same flow rate in either direction require that the geometries of the conduit on both sides of the resistance element be matched. The volume within the adapters and conduit represents a dead space for cyclic breathing. The designer, when designing the sensor, must balance the effects of decreased angle of the diffuser against the tolerable volume of dead space.

A bias flow can be used to clear this dead space. Air is drawn through the pneumotachometer from a side hole [Figure 9.4(b)] through a long tube connected to a vacuum pump. This causes a constant bias pressure drop across the pneumotachometer if the bias flow created is constant during the breathing of the patient. This approach can work well when high-frequency breathing patterns are of interest. But for low frequencies, such as those of tidal breathing, a regulating device may be required to prevent variation of the bias flow as the patient breathes.

The usable frequency range for capillary pneumotachometers is typically smaller than that for the screen type. Depending on its design, the screen-type pneumotachometer can exhibit a constant-amplitude ratio of pressure difference to flow, and zero phase shift up to as high as 70 Hz (Peslin *et al.,* 1972).

In air, the amplitude ratio of the Fleisch (capillary bundle) pneumotachometer is nearly constant at its steady-flow value up to approximately 10 Hz, increasing by 5% at 20 Hz. The phase angle between ΔP and Q increases linearly with frequency to approximately 8.5° at 10 Hz, which corresponds to an approximate 2 ms time delay between flow and pressure difference. These values change with the kinematic viscosity of the gas (Finucane *et al.,* 1972). Peslin *et al.* (1972) modeled the Fleisch pneumotachometer for frequencies up to 70 Hz by an equation of the form

$$\Delta P = RQ + L\dot{Q} \tag{9.13}$$

where L is the inertance (related to mass) of the gas in one capillary tube, defined by (7.4) and R is the flow resistance for each capillary tube, defined by (7.2). Equation (9.13) can be used as a computational algorithm to compensate the Fleisch pneumotachometer for precise measurements of a gas of constant composition at constant temperature.

EXAMPLE 9.2 A Fleisch pneumotachometer has 100 capillary tubes, each with a diameter of 1 mm and a length of 5 cm. What pressure drop occurs for a flow of 1 liter/s?

ANSWER A flow of 1 liter/s through 100 tubes is 0.00001 m^3/s through one tube. In SI units, using (7.1) and (7.2),

$$\Delta P = RF = \frac{8\eta LF}{\pi r^4} = \frac{8(0.000018)(0.05)(0.00001)}{\pi(0.0005)^4} = 367\,\text{Pa} = 3.74\,\text{cm}\,\text{H}_2\text{O}$$

An appropriately designed screen pneumotachometer may not require compensation provided by (9.13). However, it may instead be subject to equipment-generated, high-frequency noise in clinical applications. An additional point should be stressed: the frequency response of a pneumotachometer is no better than that of its associated differential pressure measurement system. It is essential that the pneumatic (acoustic) impedances, including those of the tubes and connectors between the pressure sensor and the pneumotachometer, on each side of the differential pressure sensor be balanced. This is most easily achieved by ensuring that the geometries and dimensions of the pneumatic pathways from the pneumotachometer to each side of the pressure sensor diaphragm are identical.

Equation (7.2) indicates that the resistance of the Fleisch pneumotachometer is proportional to the viscosity of the flowing gas mixture. The resistance of a screen pneumotachometer, though not computable from (7.2), is also proportional to the viscosity of the gas. The viscosity of a gas mixture depends on its composition and temperature (Turney and Blumenfeld, 1973). When inertance effects are negligible,

$$Q = \frac{\Delta P}{R(T, [F\mathrm{x}])} \tag{9.14}$$

where Q is the flow measured by the pneumotachometer for a gas mixture with species molar fractions $[F\mathrm{x}] = [N_1/N, N_2/N, \ldots, N\mathrm{x}/N]$ at absolute temperature T. Pneumotachometers are routinely calibrated for steady flow with a single calibration factor being used during experiments. Instantaneous values of T and $[F\mathrm{x}]$ are not constant during a single expiration, and their mean values change from expiration to inspiration. In particular, changes in viscosity of 10% to 15% occur from the beginning to the end of an experiment in which N_2 is washed out of the lungs by pure O_2. A continuous correction in calibration should be made when accurate results are desired.

The prevention of water-vapor condensation in a pneumotachometer is of particular importance. The capillary tubes and screen pores are easily blocked by liquid water, which decreases the effective cross-sectional area of the flow element and causes a change in resistance. Also, as water condenses, the composition of the gas mixture changes. To circumvent these problems, a common practice is to heat the pneumotachometer element, especially when more than a few consecutive breaths are to be studied. The Fleisch pneumotachometer is usually provided with an electrical resistance heater; the screen of the screen pneumotachometer can be heated by passing a current through it. In addition, heated wires can be placed inside, or heating tape or other electrical heat source can be wrapped around any conduit that carries expired gas.

Pitot Tubes The difference between stagnation pressure (measured head-on into the flow) and static pressure (measured perpendicular to the flow) in a flowing gas is the dynamic pressure, and is related to the density and the square of the velocity of the gas. Pitot tubes are flow-measuring devices based on this relationship. A modified pitot instrument has been developed that uses two pressure ports, one facing upstream and the other, downstream (Hänninen, 1991). When flow is in one direction, one port measures stagnation pressure and the other assesses static pressure. When flow reverses, the roles of the ports also reverse permitting the alternating flows of breathing to be measured. During a breath, the inspired and expired gas compositions continuously change with concomitant changes in density. Simultaneous measurement of gas composition from the same location at which the pressure measurements are made permits compensation for variations in density. Flow resistance of 1.0 cm H_2O/(liter/s), dead space of 9.5 ml, volume accuracy (from the integral of flow) of $\pm 6\%$ and flow sensitivity of 0.07 liter/s have been reported.

9.4 LUNG VOLUME

The most commonly used indices of the mechanical status of the ventilatory system are the absolute volume and changes of volume of the gas space in the lungs achieved during various breathing maneuvers. Observe Figure 9.5 and assume that a subject's airway opening and body surface are exposed to atmospheric pressure. Then the largest volume to which the subject's lungs can be voluntarily expanded is defined as the *total lung capacity* (TLC). The smallest volume to which the subject can slowly deflate his or her lungs is the *residual volume* (RV). The volume of the lungs at the end of a quiet expiration when the respiratory muscles are relaxed is the *functional residual capacity* (FRC). The difference between TLC and RV is the *vital capacity* (VC), which defines the maximal change in volume the lungs can undergo

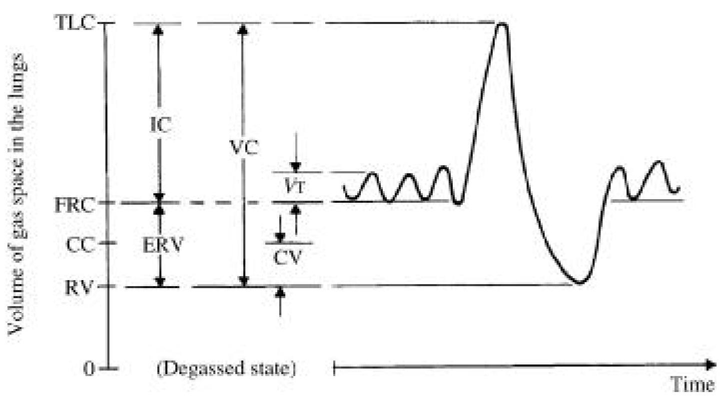


Figure 9.5 Volume ranges of the intact ventilatory system (with no external loads applied). Total lung capacity, FRC, and RV are measured as absolute volumes. Vital capacity, IC, ERV, and V_T are volume changes. Closing volume (CV) and closing capacity (CC) are obtained from a single-breath washout experiment.

during voluntary maneuvers. The vital capacity can be divided into the *inspiratory capacity* (IC = TLC − FRC) and the *expiratory reserve volume* (ERV = FRC − RV). The peak-to-peak volume change during a quiet breath is the *tidal volume* (V_T) (Petrini, 1988).

CHANGES IN LUNG VOLUME: SPIROMETRY

The measurement of changes in lung volume has been approached in two ways. One is to measure the changes in the volume of the gas space within the body during breathing by using plethysmographic techniques (discussed in Section 9.5). The second approach, referred to as *spirometry,* involves measurements of the gas passing through the airway opening. The latter measurements can provide accurate, continuous estimates of changes in lung volume only when compression of the gas in the lungs is sufficiently small. The flow of moles of gas at the airway opening can be expressed as

$$\dot{N}_{\mathrm{AWO}} = \rho_{\mathrm{AWO}} Q_{\mathrm{AWO}} \cong \rho_{\mathrm{L}} \dot{V}_{\mathrm{L}} \tag{9.15}$$

if we neglect the net rate of diffusion into the pulmonary capillary blood. This equation can be rearranged and, if the densities are essentially constant, integrated from some initial time t_0, as follows:

$$\frac{\rho_{\mathrm{AWO}}}{\rho_{\mathrm{L}}} \int_{t_0}^{t} Q_{\mathrm{AWO}}\, dt \cong \int_{t_0}^{t} \dot{V}_{\mathrm{L}}\, dt = V_{\mathrm{L}}(t) - V_{\mathrm{L}}(t_0) = v_{\mathrm{L}} \tag{9.16}$$

in which v_{L} is, according to the convention used here, the change in the volume of the lungs relative to the reference volume $V_{\mathrm{L}}(t_0)$. The density ratio accounts for differences in mean temperature, pressure, and composition that may exist between the gas mixture inside the lungs and that in the measurement sensor external to the body.

For purposes of testing pulmonary function, (9.16) is frequently implemented directly by electronically integrating the output of a flowmeter placed at a subject's mouth (with the nose blocked). However, the most common procedure for estimating v_{L}—in use since the nineteenth century—is to continuously collect the gas passing through the airway opening and to compute the volume it occupied within the lungs. This represents a physical integration of the flow at the mouth; it is performed by a device called a *spirometer.* The widespread and historical use of this device has given rise to use of the term *spirometry* to mean the measurement of changes in lung volume for testing of pulmonary function, regardless of whether a spirometer, flowmeter plus integrator, or plethysmographic technique is used. Consequently, performance recommendations have been published by the American Thoracic Society (Anonymous, 1995a) for spirometry systems in general, regardless of the primary variable measured (volume change or flow) or the type of sensor used. The recommendations address volume range and accuracy, flow range, time interval for which data are to be collected, respiratory load imposed on

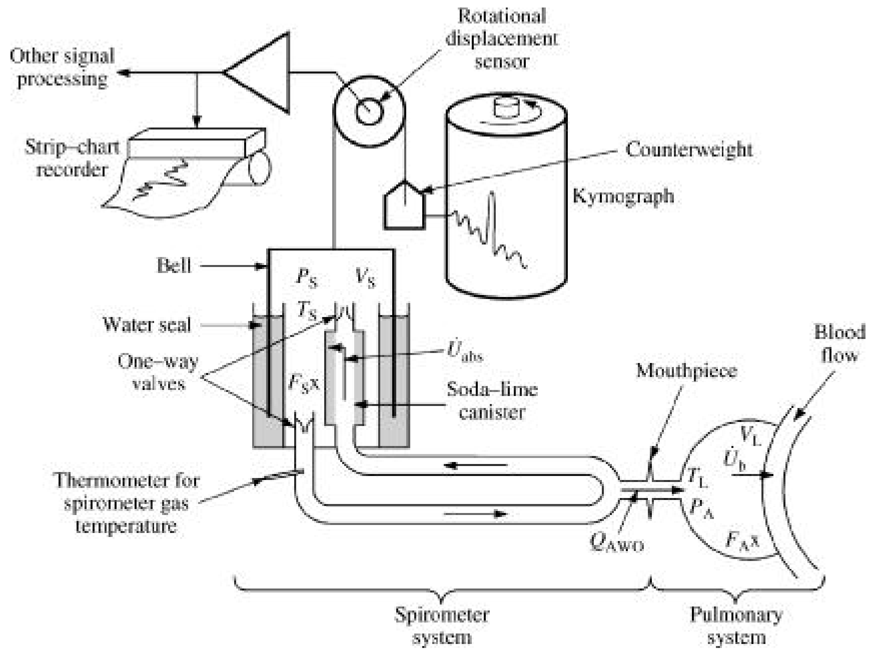


Figure 9.6 A water-sealed spirometer set up to measure slow lung-volume changes. The soda-lime and one-way-valve arrangement prevent buildup of CO_2 during rebreathing.

the subject, and calibration standards to be met by such systems for various pulmonary function tests.

A spirometer is basically an expandable compartment consisting of a movable, statically counterbalanced, rigid chamber or ''bell,'' a stationary base, and a dynamic seal between them (Figure 9.6). The seal is often water, but dry seals of various types have been used. Changes in internal volume of the spirometer, V_S, are proportional to the displacement of the bell. This motion is traditionally recorded on a rotating drum (kymograph) through direct mechanical linkage, but any displacement sensor can be used. A simple approach is to attach a single-turn, precision linear potentiometer to the shaft of the counterweight pulley and use it as a voltage divider. The electric output can then be processed or displayed.

The mouthpiece of the spirometer (Figure 9.6) is placed in the mouth of the subject, whose nose is blocked. As gas moves into and out of the spirometer, the pressure P_S of the gas in the spirometer changes, causing the bell to move. Analysis of the dynamic mechanics of a spirometer indicates that these variations in pressure are reduced by minimizing (1) the mass of the bell and counterweight and the moment of inertia of the pulley, (2) the gas space in the spirometer and tubing, (3) the surface area of the liquid seal exposed to P_S, (4) the viscous and frictional losses by appropriate choice of lubricants and type of dynamic seal, and (5) the flow resistances of any inlet and outlet tubing and valves. During resting breathing, the pressure changes in the gas within the spirometer can be considered negligible. Therefore, only the temperature, average ambient pressure, and change in volume are needed to estimate the amount of gas exchanged with the spirometer. To use the spirometer for estimates of change in lung volume during breathing patterns of higher frequency (>1 Hz) requires, in addition to the variables just mentioned, knowledge of the acoustic compliance of the gas in the spirometer (see Problem 9.3) and continuous measurement of the change of the spirometer pressure relative to ambient pressure.

The system—lungs plus spirometer—can be modeled as two gas compartments connected such that the number of moles of gas lost by the lungs through the airway opening is equal and opposite to the number gained by the spirometer. For rebreathing experiments, most spirometer systems have a chemical absorber (soda lime) to prevent buildup of CO_2. When compression of gas in the lungs and in the spirometer is neglected, mass balances on the system yield

$$\rho_L \dot{V}_L + \dot{U}_b = -\rho_S \dot{V}_S - \dot{U}_{abs} \tag{9.17}$$

The net rates of uptake from the system by the pulmonary capillary blood, $\dot{U}_b$, and the absorber, $\dot{U}_{abs}$, can be assumed constant during steady breathing. Therefore, when (9.17) is integrated with respect to time, the combined effect of these uptakes is a change in volume essentially proportional to time. This can be approximated as a linear baseline drift easily separable from the breathing pattern. Consequently, rearrangement and integration of (9.17) yield

$$v_L \cong -\frac{\rho_S}{\rho_L}(v_S - \text{drift}) = -\frac{\rho_S}{\rho_L} v_{S'} \tag{9.18}$$

from which it can be seen that the change in lung volume is approximately proportional to $v_{S'}$, the volume change of the spirometer corrected for drift.

The constant of proportionality $(-\rho_S/\rho_L)$ can be expressed in terms of the measurable quantities, pressure and temperature, by applying an equation of state to the system. With the exception of saturated water vapor, all gases encountered during routine respiratory experiments obey the ideal-gas law during changes of state:

$$P = \frac{N}{V}\mathbf{R}T = \rho \mathbf{R}T \tag{9.19}$$

Where

$\mathbf{R}$ = universal gas constant

T = absolute temperature

ρ = mole density, which, for a well-mixed compartment, equals the ratio of moles of gas, N, in the compartment to the compartment volume, V

This relationship holds for an entire gas mixture and for an individual gas species X in the mixture. For the latter, the partial pressure Px, number of moles Nx, and density $\rho x = Nx/V$ are substituted into (9.19).

A difficulty in directly substituting (9.19) into (9.18) arises from the presence of water vapor in the system. The water vapor in the lungs is saturated at body-core temperature. The water vapor in the spirometer is also saturated, even in those with dry seals, after only a few exhalations from the warmer lungs into the cooler spirometer. Saturated water vapor does not follow (9.19) during changes of state. Instead, its partial pressure is primarily a function of temperature alone.

Processes in the lungs are approximately isothermal; the change in temperature in the spirometer during most pulmonary function tests is assumed small. Therefore, we can compute the partial pressure of the ideal dry gases—the total gas mixture excluding water vapor—as the mean total pressure (taken to be atmospheric pressure P_{atm} for both the spirometer and the lungs) minus the partial pressure of saturated water vapor at the appropriate temperature. Equation (9.18) can then be evaluated as

$$v_L \cong -\left(\frac{(P_{atm} - P_{S\,H_2O})T_L}{(P_{atm} - P_{A\,H_2O})T_S}\right) v_{S'} \tag{9.20}$$

in which $P_{A\,H_2O}$ and $P_{S\,H_2O}$ are the partial pressures of saturated water vapor in the lung [47 mm Hg (6.27 kPa) at $T_L = 37\ °C$] and in the spirometer (at the measured temperature in the spirometer, T_S), respectively.

ABSOLUTE VOLUME OF THE LUNG

Because of the complex geometry and inaccessibility of the lungs, we cannot compute their volume accurately either from direct spatial measurements or from two- or three-dimensional pictures provided by various imaging techniques. Three procedures have been developed, however, that can give accurate estimates of the volume of gas in normal lungs. Two are based on static mass balances and involve the washout or dilution of a test gas in the lungs. The test gas must have low solubility in the lung tissue. That is, movement of the gas from the alveoli by diffusion into the parenchyma (tissue) and blood must be much less than that occurring by convection through the airways during the experiment. The third procedure is a total body plethysmographic technique employing dynamic mass balances and gas compression in the lungs (see Section 9.5). These estimates of lung volume provide a static baseline value of absolute lung volume that can be added to a continuous measure of the change of lung volume to provide a continuous estimate of absolute lung volume.

The computational formulas for the test-gas procedures described below are routinely derived in the literature in terms of a volume fraction of the test gas in a mixture. The volume fraction of a gas X is an alternative expression of, and is numerically equal to, its molar fraction, F_X. When the ideal-gas law (9.19) is applied to X alone and to the total mixture containing X, we can express F_X in terms of the partial pressure of X, P_X.

$$F_X = \frac{N_X}{N} = \left(\frac{P_X V}{\mathbf{R}T}\right)\left(\frac{\mathbf{R}T}{PV}\right) = \frac{P_X}{P} \tag{9.21}$$

This follows from Dalton's law of partial pressures, for which all gases in a mixture are visualized as having the same temperature and occupying the same volume. On the other hand, the concept of volume fraction lends itself to the use of the spirometer to measure the volume occupied by a mass of gas at a given pressure and temperature. Assume that $N\mathrm{x}$ moles of X at temperature T occupy a volume $V\mathrm{x}$ when subjected to a pressure P. These $N\mathrm{x}$ moles of X are then added to an X-free gas mixture so that the total moles of mixture is N, and the mixture is allowed to occupy volume V at pressure P. Applying the ideal-gas law to X before addition to the mixture and to the mixture after the addition of X, we can evaluate $F\mathrm{x}$ for the final mixture as

$$F\mathrm{x} = \frac{N\mathrm{x}}{N} = \left(\frac{PV\mathrm{x}}{\mathbf{R}T}\right)\left(\frac{\mathbf{R}T}{PV}\right) = \frac{V\mathrm{x}}{V} \tag{9.22}$$

Thus the molar fraction $F\mathrm{x}$ can be thought of either as a partial-pressure fraction (9.21), or as an equivalent-volume fraction (9.22) in which the volumes are those that the components of the gas or the gas mixture would occupy if all exhibited the *same* temperature and total pressure. However, in some experiments the temperature and/or total pressure changes from one measurement to another. Remember that in the mass balances describing such experiments, $F\mathrm{x}$ represents molar fraction and must be evaluated as such to account for the observed changes in temperature and pressure. Instruments capable of measuring molar fractions and concentrations of gases in a mixture are described in Section 9.7.

NITROGEN-WASHOUT ESTIMATE OF LUNG VOLUME

Figure 9.7 is a diagram of an apparatus that can be used during a multi-breath N_2 washout. The subject inhales only an N_2-free gas mixture (for this example, O_2) because of the one-way valves, but she or he exhales N_2, O_2, CO_2, and water vapor. This experiment is routinely performed to measure FRC, so the subject is switched into the apparatus at the end of a quiet expiration following normal breathing of atmospheric air. He or she is allowed to breathe the N_2-free mixture in a relaxed manner around FRC with relatively constant tidal volumes for a fixed period of time (7 to 10 min) or until the N_2 molar fraction in the expirate is sufficiently near zero ($<2\%$).

A static mass (molar) balance on the N_2 in the lungs from before to after the washout yields an estimate of the lung volume at which the first inspiration of O_2 began. Assume that during the experiment, negligible amounts of N_2 diffuse into the alveolar gas from lung tissue and pulmonary capillary blood. Therefore, the change in the number of moles of N_2 in the lungs as a result of the washout is just the number lost during each expiration (and gained by the spirometer). Assuming that measurements of N_2 fraction are made on a

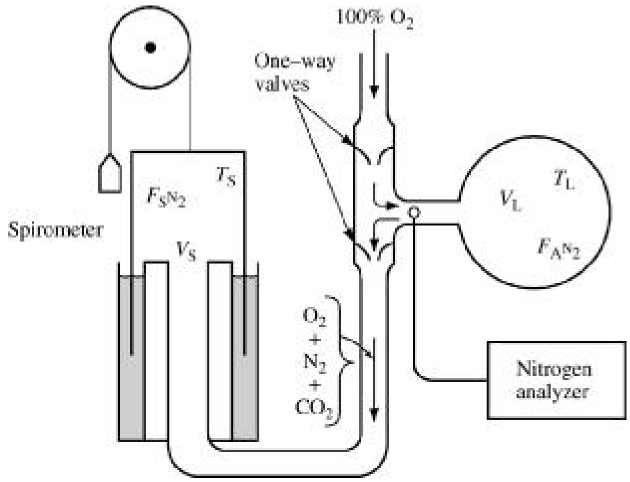


Figure 9.7 Diagram of an N_2-washout experiment The expired gas can be collected in a spirometer, as shown here, or in a rubberized-canvas or plastic Douglas bag. N_2 content is then determined off line. An alternative is to measure expiratory flow and nitrogen concentration continuously to determine the volume flow of expired nitrogen, which can be integrated to yield an estimate of the volume of nitrogen expired.

wet-gas basis and that the spirometer contained no N_2 at the start of the experiment, we find that a static mass balance yields

$$F_{AN_2}(t_1)\frac{V_L(t_1)}{T_L} - F_{AN_2}(t_2)\frac{V_L(t_2)}{T_L} = F_{SN_2}(t_2)\frac{V_S(t_2)}{T_S}$$

If the lung volume at the beginning of the washout, t_1, is the same as that at the finish, t_2, then the equation can be rearranged as follows:

$$V_L = \frac{T_L}{T_S}\left(\frac{F_{SN_2}(t_2)V_S(t_2)}{F_{AN_2}(t_1) - F_{AN_2}(t_2)}\right) \tag{9.23}$$

When this volume is that at the end of a quiet expiration, V_L is the FRC.

All variables on the right-hand side of (9.23) are measurable. The numerator in the brackets in (9.23) represents the equivalent expired N_2 volume for the conditions in the measuring device (the spirometer in Figure 9.7). However, if the nitrogen molar fractions in (9.23) were to be measured on a dry-gas basis rather than on a wet basis as assumed here, the right-hand side would have to be multiplied by the dry-gas partial-pressure ratio $(P_{atm} - P_{SH_2O})/(P_{atm} - P_{AH_2O})$.

EXAMPLE 9.3 In an N_2-washout experiment, the subject's cumulative expired volume into a spirometer is 5 liters. At the beginning of the experiment, the spirometer has a volume of 7 liters but contains no N_2. At the end of the experiment, the molar fraction of N_2 in the spirometer is 0.026 and the $F_{A}N_2$ of the subject has decreased by 0.1. The final temperature of the spirometer is 303 K. What was the lung volume at which the subject was breathing?

ANSWER $V_S(t_2) = 5$ liters + 7 liters, $F_SN_2(t_2) = 0.026$, $F_AN_2(t_1) - F_AN_2(t_2) = 0.1$, $T_S = 303\,\text{K}$, $T_L = 310\,\text{K}$

$$V_L = \frac{T_L}{T_S}\left(\frac{F_SN_2(t_2)V_S(t_2)}{F_AN_2(t_1) - F_AN_2(t_2)}\right) = \frac{310}{303}\left(\frac{0.026 \times 12}{0.1}\right) = 3.19\,\text{liter}$$

A washout experiment of this type gives an estimate of the volume of the gas space in the lungs that freely communicates with the airway opening. This is the entire alveolar volume in normal young, adult lungs. In diseased lungs, in which airways are totally or partially obstructed (by mucus, edema, bronchospasm, or tumors, for instance), this estimate of the gas volume in the lungs at FRC can be low.

HELIUM-DILUTION ESTIMATE OF LUNG VOLUME

The procedure described in the previous paragraph involves the removal from the lungs of gas normally resident there in known concentration. An alternative approach is to add a measured amount of a nontoxic, insoluble *tracer* gas to the inspirate, and—after it is uniformly distributed in the lungs—determine its concentration and compute lung volume. Helium is often used as the tracer gas for this experiment, though argon and neon are also acceptable.

The lungs communicate directly with the spirometer, as shown in Figure 9.6. At the beginning of the experiment, a fixed amount of He is added to the spirometer. The amount of He added is routinely measured as an equivalent volume at the initial conditions in the spirometer. The change in volume of the spirometer due to addition of pure helium, $V_S\text{He}$, divided by the total spirometer volume after the addition of the helium, $V_S(t_1)$, is $F_S\text{He}(t_1)$, the molar fraction of helium on a wet-gas basis in the spirometer at the beginning of the experiment [see (9.22)].

The subject is typically allowed to begin rebreathing from the spirometer when the lung volume is at FRC. The subject breathes at his or her resting rate and tidal volume until the concentration of He in the lungs is in equilibrium with that in the spirometer—that is, until $F_A\text{He}(t_2) = F_S\text{He}(t_2)$. The tracer-gas molar fraction in the expirate is continuously measured by withdrawing a small stream of gas from the mouthpiece, passing it through a He analyzer, and returning it to the spirometer. Equilibration is judged to have occurred when the change in the He fraction from one breath to the next is arbitrarily small. As equilibration approaches, the mean spirometer volume is maintained at its original volume $V_S(t_1)$ by the addition of O_2 if required. The experiment is terminated at the end of a quiet expiration (FRC).

The He is redistributed between spirometer and lungs during rebreathing, but the total amount remains essentially constant because no appreciable quantities are lost by diffusion into the tissues. The average total number of moles of dry gas in the system is kept constant by chemically removing, via the soda-lime canister, the CO_2 added from the blood, and replenishing the O_2 taken up. Therefore, the system can be considered closed with respect to the test gas only, and a static mass (molar) balance for He may be written as

$$F_S\text{He}(t_1)\frac{V_S(t_1)}{T_S(t_1)} = F_S\text{He}(t_2)\frac{V_S(t_2)}{T_S(t_2)} + F_A\text{He}(t_2)\frac{V_L(t_2)}{T_L}$$

for molar fractions measured on a wet basis. This can be rearranged and evaluated when the He fractions in the lungs and spirometer are equal.

$$V_L = \frac{V_S(t_1)}{F_S\text{He}(t_2)}\left[\frac{T_L}{T_S(t_1)}F_S\text{He}(t_1) - \frac{T_L}{T_S(t_2)}F_S\text{He}(t_2)\right] \tag{9.24}$$

V_L is an estimate of FRC for the experiment performed as we have described. Equation (9.24) is frequently rewritten in terms of the equivalent volume of He originally added to the system, $V_S\text{He}$.

$$V_L = \frac{T_L}{T_S(t_1)}\left(\frac{V_S\text{He}}{F_S\text{He}(t_2)}\right) - \frac{T_L}{T_S(t_2)}\left(\frac{V_S\text{He}}{F_S\text{He}(t_1)}\right) \tag{9.25}$$

If the molar fractions are measured on a dry basis, the temperature ratios in (9.24) and (9.25) must be multiplied by their corresponding dry-gas partial-pressure ratios: $[P_{atm} - P_S\text{H}_2\text{O}(t_1)]/(P_{atm} - P_A\text{H}_2\text{O})$ and $[P_{atm} - P_S\text{H}_2\text{O}(t_2)]/(P_{atm} - P_A\text{H}_2\text{O})$, respectively.

The volume computed from (9.24) and (9.25) is a measure of the volume of gas space in the lungs for which the final He molar fraction in the spirometer, $F_S\text{He}(t_2)$, is a representative value. If some parts of the lung do not communicate freely with the airway opening, as in obstructive lung disease, then these equations can provide a low estimate of the FRC.

9.5 RESPIRATORY PLETHYSMOGRAPHY

The term *plethysmography* refers, in general, to measurement of the volume or change in volume of a portion of the body. In respiratory applications, plethysmography has been approached in two ways: by inferring changes in thoracic-cavity volume from geometrical changes at discrete locations on the torso and by measuring the effects of changes in thoracic volume on variables associated with the gas within a total-body plethysmograph.

THORACIC PLETHYSMOGRAPHY

Several devices have been used to measure continuously the kinematics (motions) of the chest wall that are associated with changes in thoracic volume

Modeling and Experimentation: Mass-Spring-Damper System Dynamics

Prof. R.G. Longoria

Department of Mechanical Engineering
The University of Texas at Austin

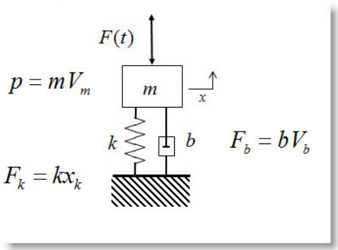
October 21, 2014

Overview

1. Review two common mass-spring-damper system models and how they are used in practice
2. The standard linear 2nd order ODE will be reviewed, including the natural frequency and damping ratio
3. Show how these models are applied to practical vibration problems, review lab models and objectives
4. Discuss methods that can be helpful in improving estimates of the system parameters, introduce the log decrement method for measuring damping ratio

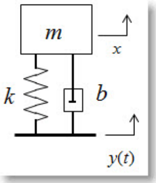
Most common mass-spring-damper models

Fixed-base configuration, spring and damper in parallel.



Ex: structures, buildings, etc.

Base-excited configuration, spring and damper in parallel, motion input at base..



Ex: Vehicle suspensions, seismic sensors

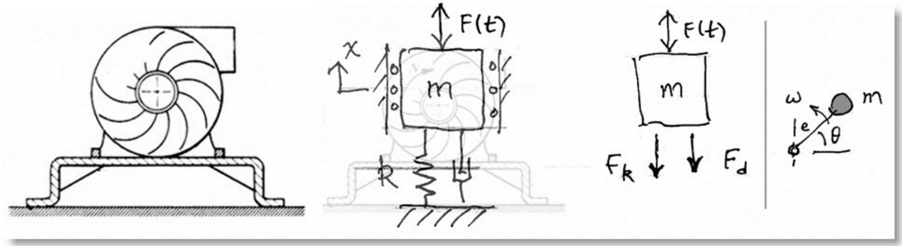
When a parameter like k or b is indicated, it usually implies that a *linear* constitutive law is implied for that model element.

Using mass-spring-damper models

1. Use these models to represent a wide range of practical situations, not just translation but rotation. The models have analogies in all other energy domains.
2. Recognize 'forcing' in each case: force $F(t)$ on mass for fixed-based compared with velocity $\dot{y}(t)$ at the base for base-excited system
3. Unforced response. Some problems are concerned with the system responding to initial conditions and *no forcing* (i.e., $F(t) = 0$, $\dot{y}(t) = 0$), so the transient response and the eventual rest condition is of interest.
4. Forced response. If there is forcing, there may be a need to understand transient changes and then the 'steady' operation under forcing. Types include: step, impulse, harmonic, random

Example: unbalanced fan on support base

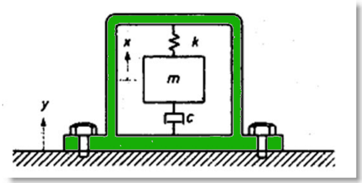
An unbalanced fan is mounted on a support base, and the model to study induced vibration is illustrated below. This is a fixed-base configuration.



An additional modeling issue in this problem is coming up with the forcing function, $F(t)$, which would come from understanding how the eccentricity in the fan impeller induces a force on the total fan mass. Note an assumption can also made that the fan only vibrates in the vertical direction, if the base structure is very stiff laterally. All of these decisions are part of the modeling process.

Example: motion sensor (seismic sensor)

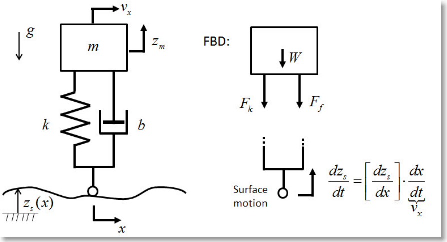
An example of a system that is modeled using the based-excited mass-spring-damper is a class of motion sensors sometimes called seismic sensors. Accelerometers belong to this class of sensors.



The spring and damper elements are in mechanical parallel and support the 'seismic mass' within the case. The case is the base that is excited by the input base motion, $y(t)$.

Example: suspended mass moving over a surface

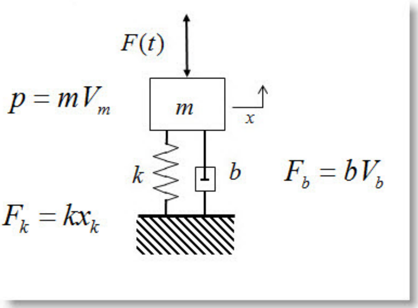
The diagram below is uses a base-excited configuration to model a mass moving over a surface. The spring and damper elements might represent, for example, the tire contacting the ground as the vehicle moves in the x direction. Vibration of the mass is in the z direction.



An automotive suspension model like this would represent only a quarter of the vehicle, and there would be another *stage* that represents the actual suspension. Note how you model the *base-motion* by factoring how fast you move over a ground profile, $z_s(x)$, a function of distance traveled, x.

Model of fixed-base mass-spring-damper system

Consider the fixed-base system below.



Applying Newton's law to a free-body diagram of the mass,

$$m\ddot{x} = \sum F = -F_b - F_k + F,$$

$$m\ddot{x} = -b\dot{x} - kx + F,$$

Rearrange to derive a 2nd order ODE,

$$m\ddot{x} + b\dot{x} + kx = F.$$

Now, in standard form,

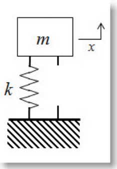
$$\ddot{x} + \underbrace{\left[\frac{b}{m}\right]}_{\triangleq 2\zeta\omega_n}\dot{x} + \underbrace{\left[\frac{k}{m}\right]}_{\triangleq \omega_n^2} x = u$$

where $u = F/m$, ω_n is the *undamped* natural frequency, and ζ is the damping ratio.

From the undamped natural frequency, define the undamped natural period, $T_n = 2\pi/\omega_n$.

Undamped harmonic motion, $\zeta = 0$

For the unforced case, $F = 0$, and undamped, $b = 0$, case,



the model equation becomes, $\ddot{x} + \omega_n^2 x = 0$, with a complete solution for the displacement of the mass,

$$x = x_o \cos(\omega_n t) + \frac{\dot{x}_o}{\omega_n} \sin(\omega_n t)$$

where x_o is the initial displacement and $\dot{x}_o$ the initial velocity of the mass.

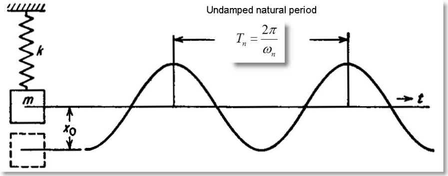
This solution gives x for all time given only the initial conditions and the natural frequency, which in this case is $\omega_n = \sqrt{k/m}$.

NOTE: This ideal case is a 'linear oscillator', a model used to described many other types of systems such as the simple or compound pendulum in small motion. We can see how ω_n in those cases would contain parameters such as length, gravity, mass.

Undamped harmonic motion

If you give the mass an initial displacement x_o and release from rest ($\dot{x}_o = 0$), then this case predicts it would oscillate indefinitely,

$$x = x_o \cos(\omega_n t)$$



Given $x(t)$, the velocity and acceleration are readily found:

$$v = -x_o \omega_n \sin(\omega_n t)$$

$$a = +x_o \omega_n^2 \cos(\omega_n t)$$

NOTE: The undamped natural frequency directly influences peak values of velocity and acceleration.

What about the effect of gravity?

The axis of mass motion may be in line with the gravity vector, so sometimes it is necessary to consider the effect of the force due to gravity, mg.

In vibration problems, however, especially when all the elements follow linear constitutive laws (i.e., spring, $F = kx$, damper, $F = bV$), the effect of gravity will not influence the dynamics. Only the *equilibrium* position of the mass will be changed.

If you had a spring that was not linear, then it is important to consider gravity because vibrational motion will be influenced by the spring behavior in a nonlinear manner.

Bottom line: If system is linear, use gravity to find where your system 'sits' initially, but then the linear solutions discussed here apply directly for motion about that point. If nonlinear, you may need to follow other approaches such as linearizing about the equilibrium, which is found by considering gravity. Direct numerical simulation of the system is often an alternative.

Consider the damped cases now, $\zeta \neq 0$

The special *undamped* case has been described. For systems where $b \neq 0$, the damping ratio will not be zero. Solutions for these cases are classified by ζ, and a system is:

- *underdamped* if $\zeta < 1$,
- *overdamped* if $\zeta > 1$,
- *critically damped* if $\zeta = 1$

The solutions are known for these cases, so it is worthwhile formulating model equations in the standard form,

$$\ddot{x} + 2\zeta\omega_n\dot{x} + \omega_n^2 x = u(t)$$

Detailed derivations can be found in system dynamics, vibrations, circuits, etc., type textbooks. Selected excerpts will be posted on the course log for reference.

Underdamped motion, $\zeta < 1$

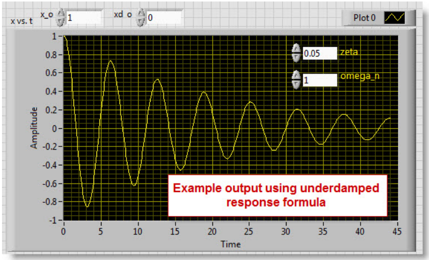
If you give the mass an initial displacement x_o and initial velocity $\dot{x}_o$, the solution for the response, x, is,

$$x = e^{-\zeta\omega_n t}\left[\frac{\dot{x}_o + \zeta\omega_n x_o}{\omega_d}\sin(\omega_d t) + x_o\cos(\omega_d t)\right], 0 < \zeta < 1$$

where $\omega_d = \omega_n\sqrt{1-\zeta^2} = 2\pi/T_d$ is the *damped* natural frequency, and T_d the damped natural period.

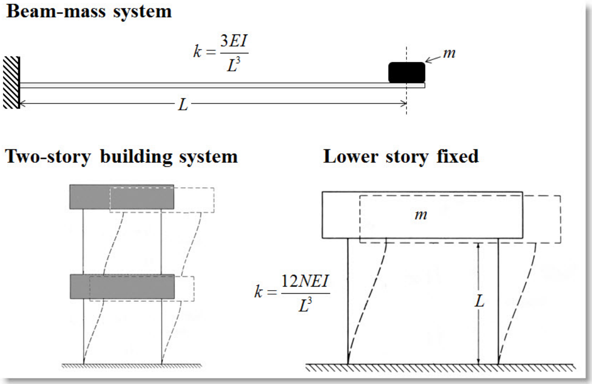
Given $x(t)$, the velocity and acceleration can be found by differentiation.

NOTE: The damped natural frequency is dependent on both the undamped natural frequency and the damping ratio.



Possible lab models

Demonstrating the mass-spring-damper model in the lab can be done in several ways. Simply suspending a mass with a spring-like element is easy, but it can be difficult to get purely one-degree-of-freedom motion. Using beam-mass combinations can provide a simple and reliable way to test the concepts we've discussed.

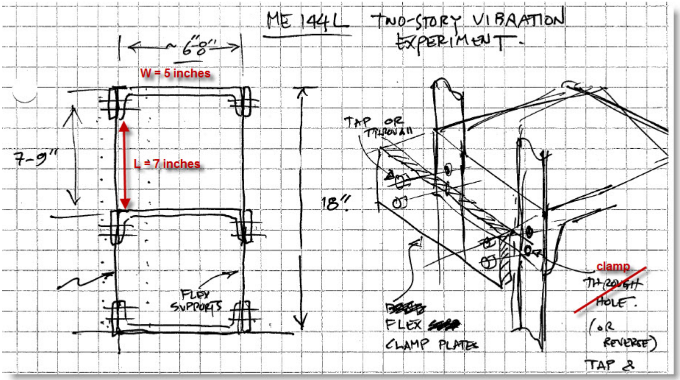


Typical laboratory objectives

1. Estimate the mass and stiffness of the system components before going to lab, and verify or improve once in lab. Try to understand sources of error.
2. Use measurements of motion to estimate the effective system damping ratio, ζ, which can be used to estimate effective damping, b, if the system is linear.
3. Determine the damped and undamped natural frequencies.
4. Assess accuracy of system parameter values.
   - Use the measured natural frequency to estimate the stiffness, assuming confidence in mass estimate.
   - Assess model by comparing theoretical versus measured response. For example, compare measured acceleration to that predicted by model response.
5. Estimating displacement from accelerometer measurement can be challenging. Explore this by integrating the acceleration to get displacement.

Two-story building lab model

The concept for the two-story lab model is shown below. The flex guide lengths, L, are nominally 7 inches, but these can be adjusted. The actual ‘floors’ are 5 inches in width (and depth), made from aluminum U-channel.



The specific material list is provided on the course log.

How well can you estimate ω_n?

Once you get into the lab, you can measure the mass and stiffness to improve the theoretical estimate of ω_n. How accurate is this value?

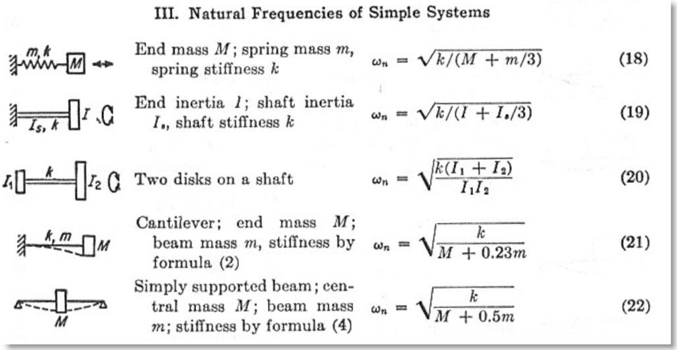
A comparison made to a measurement of ω_n is preferred, however remember that we cannot directly measure this value from $T_n = 2\pi/\omega_n$ except in the undamped case. Only the *damped* natural frequency, T_d, can be measured. Since,

$$T_d = \frac{2\pi}{\omega_d} = \frac{2\pi}{\omega_n\sqrt{1-\zeta^2}},$$

you can only estimate ω_n if you have an estimate of ζ, which can be done using the log decrement method. Before discussing that method, first consider some sources that could influence estimates of ω_n.

Influence of spring mass on undamped natural frequency

From appendix in Den Hartog [1] (see handout on course log), the undamped natural frequency for several simple 'mass-spring' systems can be estimated *theoretically* (using Rayleigh's method [2]) as follows:



III. Natural Frequencies of Simple Systems

m, k; M	End mass M; spring mass m, spring stiffness k	$\omega_n = \sqrt{k/(M + m/3)}$	(18)
I_s, k; I	End inertia I; shaft inertia I_s, shaft stiffness k	$\omega_n = \sqrt{k/(I + I_s/3)}$	(19)
I_1; k; I_2	Two disks on a shaft	$\omega_n = \sqrt{\frac{k(I_1 + I_2)}{I_1 I_2}}$	(20)
k, m; M	Cantilever; end mass M; beam mass m, stiffness by formula (2)	$\omega_n = \sqrt{\frac{k}{M + 0.23m}}$	(21)
M	Simply supported beam; central mass M; beam mass m; stiffness by formula (4)	$\omega_n = \sqrt{\frac{k}{M + 0.5m}}$	(22)

Note that a *fractional* mass of the spring element is used in each formula.

Bounds on undamped natural frequency estimate

The influence of spring mass suggests one way to calculate upper and lower bounds on the undamped natural frequency is to consider:

Case 1: Assume spring is *massless*. In this case, the undamped natural frequency is,

$$\omega_{n1} = \sqrt{k/m}.$$

Case 2: Assume all the spring mass, m_s, is lumped into main mass. In this case, the undamped natural frequency is,

$$\omega_{n2} = \sqrt{k/(m+m_s)}.$$

The *actual* undamped natural frequency has to lie between these two values.

$$\omega_{n2} \leq \omega_{n,\text{actual}} \leq \omega_{n1}$$

This assumes you know the stiffness well. Of course, one could do a complete uncertainty analysis on both parameters, but if you can assume a good estimate of one or the other these quick bounds can be easily calculated.

Experimental determination of damping ratio (1)

From the model for underdamped response of an *unforced* second order system presented earlier,

$$x = e^{-\zeta\omega_n t}\left[\frac{\dot{x}_o + \zeta\omega_n x_o}{\omega_d}\sin(\omega_d t) + x_o\cos(\omega_d t)\right], 0 < \zeta < 1$$

which can be expressed,

$$x(t) = A_o e^{-\zeta\omega_n t}\cos\left[\omega_d t - \phi\right]$$

where,

$$A_o = \left[\left(\frac{\dot{x}_o + \zeta\omega_n x_o}{\omega_d}\right)^2 + x_o^2\right]^{1/2} \quad \text{and} \quad \tan\phi = \frac{\dot{x}_o + \zeta\omega_n x_o}{\omega_d x_o}$$

The important result here is that the amplitude of the response follows a *decaying exponential* that is a function of initial conditions, ζ, and ω_n.

Experimental determination of damping (2)

The amplitude decay can be determined using peak values measured every T_d seconds every cycle, n. The amplitude ratio between successive peaks is then,

$$\frac{A_n}{A_{n+1}} = \exp\left[\zeta\omega_n\left(t_{n+1} - t_n\right)\right]$$

and the period between peaks is,

$$\left(t_{n+1} - t_n\right) = T_d = 2\pi/\omega_d = \frac{2\pi}{\omega_n\sqrt{1-\zeta^2}}.$$

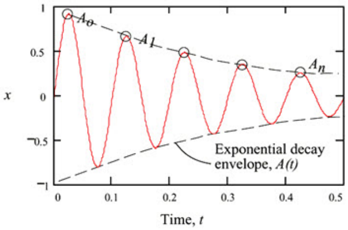
Now, taking log of both sides of the amplitude ratio relation,

$$\ln\left[\frac{A_n}{A_{n+1}}\right] = \left[\frac{2\pi\zeta}{\sqrt{1-\zeta^2}}\right]\cdot n$$

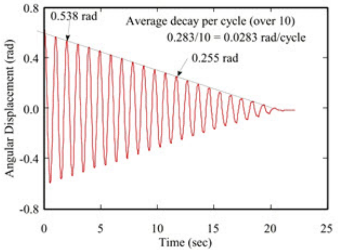
This shows that log of the amplitude ratios–the log decrement–is *linearly* related to the cycle number, n, by a factor that is a function of ζ.

Example decay envelopes

Exponential decay:



Linear decay:

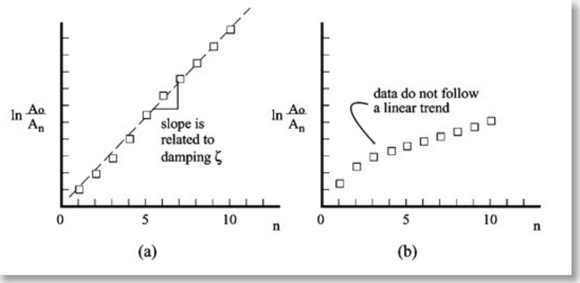


An envelope that takes on an exponential shape may suggest linear damping is dominant, while the envelope on the right with linear decay indicates the system does not have linear damping.

Log decrement quantifies the damping

The relation revealed from the log decrement data will suggest if a system has dominant linear damping.

Case (a) on the left indicates linear damping, while case (b) is for a system with Coulomb (nonlinear) damping, for which a constant ζ is not defined.



Note that the slope of the log decrement plot is, $\beta = \left[\frac{2\pi\zeta}{\sqrt{1-\zeta^2}}\right]$, from which ζ can be found.

Summary

- Study of mass-spring-damper system models provides insight into a wide range of practical engineering problems.
- You can understand the underlying design of many types of sensors such as accelerometers by understanding 2nd order system dynamics.
- In lab, we will use an accelerometer to measure acceleration. A related lecture reviews practical use of accelerometers.
- The lab work in this first week should provide the basis for building a model for the full two-story system.

References

[1] J.P. Den Hartog, Mechanical Vibrations, Dover Publications, New York, 1934

[2] W.T. Thomson, Theory of Vibrations with Applications, 4th ed., Prentice-Hall, 1993